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Selected Applications of a Biaxial Tiltmeter in the Ground Motion Environment

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The features of the biaxial tiltmeter that allow its utilization for monitoring various seismic phenomena are discussed. Long-term earthtide level data are presented comparing the sensitivity threshold and stability with laboratory standard mercury tube tiltmeters. Use of the device for earthquake prediction capability is explored through presentation of data showing precursor tilts for a large number of seismic events.

Introduction

DESIGN characteristics and applications of the two-axis electrolytic bubble level as a primary vertical reference in missile systems have been presented in a previous paper.[1] Recent major innovations in fabrication techniques made with respect to the tilt sensor have a strong bearing upon its integrity as a standard instrument for measuring seismic phenomena. It is deemed appropriate, therefore, to mention some of the innovations and elaborate a little more fully on data that can provide insight into the capability of the instrument. When measuring data in the earthtide amplitude range (1 μrad), it is necessary for one to have a comprehensive understanding of the characteristics of the instruments to be utilized as well as some feeling for the environment in which the instruments are to perform. Clearly, the problem of defining instrument drift characteristics on a time scale consistent with the requirements of "earthtide" level measurements poses a great difficulty. If a high confidence level can be established in the drift and reliability characteristics of the instrument itself based upon sound prior investigation, then the acceptability of the measurement data will be maintained, though it may not conform to an expected behavioral pattern. It is recognized that data from multiple instruments located in close proximity to each other often provides a means for separating instrument error from the effects being measured, but cost considerations do not often allow the luxury of having the described redundancy feature, especially at remote seismic field installations such as utilized in earthquake prediction studies and volcano activity studies. In such studies, the inherent cost for a large number of instruments to map the area of interest precludes the use of more than one instrument in each site except in unusual circumstances. Thus, it becomes worthwhile for considerable effort to be devoted toward proving the instrument prior to its acceptance as a dependable device for use in the various projects where tiltmeter data are deemed useful. Because the sensing element of the tiltmeter was developed initially for military purposes, the focus of this development effort was upon long-term reliability. Thus, in shifting the application for the device over to a nonmilitary role, the reliability feature was a built-in bonus for these other applications. The role of proving the tiltmeter for use and acceptance as an instrument for eathquake fault monitoring and volcano erruption studies has been left to the using agencies such as the U.S. Geological Survey and California Division of Mines and Geology since they are the primary commercial customers to first utilize the device in this nonmilitary role. Discussions to follow will cover techniques and data utilized to prove the instrument and to gain a knowledge

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Index categories: LV/M Guidance Systems; Research Facilities and Instrumentation.

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of ways to interface with the surrounding geological environment for effective installation of the tiltmeter.

Description of the Tiltmeter

The tiltmeter as presently configured, includes a sensor package in a special mount connected to an electronics module. The special mount seals against moisture, where applicable, attenuates thermal gradients, provides the required mechanical attachment to the environment being measured, and preserves the stability and precision of the sensor. The tiltmeter electronics perform the functions of impedance matching, amplifying, demodulating, and filtering both axes of the sensor outputs to provide biaxial tilt signals in analog form. Depending on the application requirements, these analog signals are suitable for the following: 1) meter and recorder inputs to indicate off-level conditions or relative motions; 2) to provide signals for telemetry equipment for remote monitoring; and 3) to drive switching circuits that provide binary information to digital computers.

The electronics circuits for the tiltmeter have been designed using criteria adopted from Minuteman reliability concepts where MTBFs of a few million hours are not uncommon. The calculated MTBF for the nonmilitary tiltmeter electronics is 8600 hr or approximately 10 yr.

The heart of the tiltmeter is the sensor itself. The uniqueness of this device lies in both the choice of materials selected for its fabrication and proprietary processes utilized for controlling cleanliness during fabrication. Extreme caution is taken to prevent contaminating elements from being introduced during processing of the precision optical parts that comprise the sensor. Further, quality control is exercised with respect to particle removal in all flushing and filling fluids introduced into the assembled sensor. Nothing must be present that will deteriorate the normal surface wetting characteristics at the electrolyte-glass interface within the sensor. Experience has shown that a satisfactorily cleaned sensor will perform extremely well, whereas a contaminated sensor will often show signs of instability.

The biaxial tiltmeter has both a borehole configuration and a tilt-table version. The borehole configuration shown in Fig. 1 is designed for applications where the installation is reasonably permanent and where the benefits of a subterranean thermal environment are desired. The tilt-table version shown in Fig. 2 is more suitable where the instrument needs to be portable and where more frequent adjustments may be required. Both configurations utilize the same sensor and the same electronics package.

Tiltmeter Resolution Limits

The advent of geophysical requirements for earth tilt measurements in the order of 10^{-9} rad per day led to a consideration of the theoretical threshold capability of the tiltmeter sensor. A survey of previous test measurements made at Autonetics indicated that instrument measurement variability errors and environmentally induced errors were in the neighborhood of 0.5 arc sec rms (2.5×10^{-6} rad). It became obvious that data at hand yielded no means for establishing the tilt sensing threshold which was expected to be significantly below the 0.5 arc sec range. Early in 1972, a program was instituted to determine this threshold sensitivity as well as to establish more specific data about short- and long-term drift rates. An analysis was accomplished which computed the theoretical limits for both the intrinsic tilt noise threshold of the sensor and the associated tilt noise threshold for the detection electronics. The predominant random noise limitations on the tiltmeter were determined to be the following: 1) Brownian random motion of the sensor bubble; 2) thermal noise in the bridge resistors; and 3) equivalent angular input noise current of the bridge amplifier.

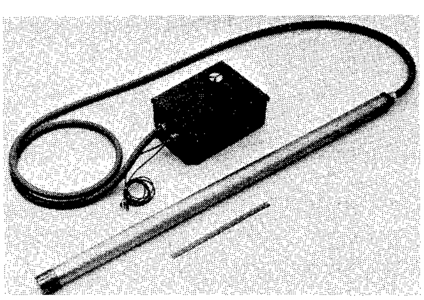

Fig. 1 Autonetics biaxial borehole tiltmeter.

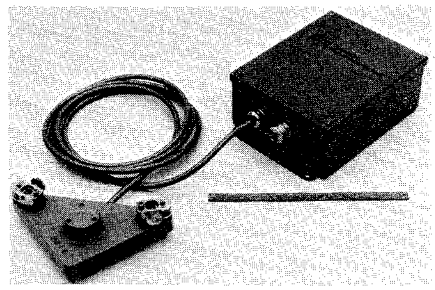

Fig. 2 Biaxial tiltmeter, tilt table version.

Derivation of the random force function for the equation of motion of the bubble is shown in the Appendix, and the rms deviation θB(RMS) determined from this derivation is as follows:

$$\theta_{B(RMS)} = 3.1 \times 10^{-9}\,\mathrm{rad/Hz^{1/2}} \quad (1)$$

Thermal noise in the bridge resistors and the equivalent angular input noise current of the bridge amplifier gave a resultant random noise referred to tilt angle of the following:

$$\theta_{E(RMS)} = 6.0 \times 10^{-9}\,\mathrm{rad/Hz^{1/2}} \quad (2)$$

The resultant value of these two random signals yields a noise equivalent threshold for the tiltmeter circuit of the following:

$$\theta_{(RSS)} = 6.8 \times 10^{-9}\,\mathrm{rad/Hz^{1/2}} \quad (3)$$

The test setup for empirically determining threshold sensitivity and short-term drift rates consisted of two sensors with low-noise, high-gain detection electronics having a scale-factor of 1 mv/10^{-9} rad. Detector electronics consisted of a resistance bridge differential amplifier, a midamplifier, a 4.8 kHz full wave synchronous demodulator, and a d.c. amplifier and low-pass active filter, with adjustable bandwidth from 1.0 Hz to 0.03 Hz. The narrow band reduced microseismic disturbances, limited the electronic white-noise contribution, and allowed measurements of Brownian motion and short-term drifts. A controlled and monitored thermal environment was provided as thermal isolation from ambient effects. The sum and differences of the two sensors were measured to separate random phenomenon from systematic phenomenon. Measured results for the noise equivalent tilt thresholds ranged from a maximum sensitivity of 1.5×10^{-9} rad for a bandwidth of 0.025 Hz to a threshold sensitivity of 1.0×10^{-8} rad for a bandwidth of 1.25 Hz. These results are slightly larger than the computed noise thresholds (1.5 larger), but appear to follow the square root of the bandwidth function closely. Figure 3 compares noise equivalent threshold vs measurement bandwidth.

Tiltmeter Drift Characteristics

Short-term drift rates taken from the same series of data described above were consistent with those which can be expected at the various bandwidths. Smoothed data would indicate that the drift rate improves with smoothing time, as would be expected.

Long-term drift having the highest degree of resolution yet obtained has come from a Rockwell International tiltmeter

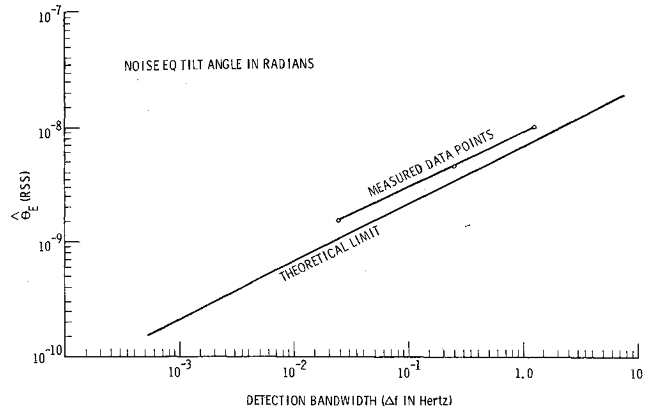

Fig. 3 Noise equivalent threshold of biaxial tiltmeter.

installed for test at the U.S. Geological Survey Vault at the Presideo in San Francisco. The installation was made the first week of April 1972 and the data from this test is shown in Fig. 4. The tiltmeter utilized was similar to the tilt-table configuration depicted in Fig. 2. It was installed in an isothermal environment within the vault with the axes aligned collinear with two single axis mercury tube tiltmeters also mounted within the vault. The Y-axis of the biaxial tiltmeter was oriented in a north–south direction a few feet away from one of the 5 m mercury tube tiltmeters (No. 2) and nearly collinear with its axis. The X-axis of the biaxial tiltmeter was oriented in an east–west direction nearly collinear with mercury tube tiltmeter No. 1 located in another wing of the vault within a radial distance of 35 ft.

The particular biaxial tiltmeter evaluated had a scale factor of 200 mv/μrad, but data was recorded with the recorder pen sensitivity reduced to $\frac{2}{3}$ that of the mercury tube tiltmeter in order to allow it to remain on the recorder chart while thermal stabilization and settling occurred.

Figure 4 is a tidal level record covering 56 days of data from April 7, 1972 to June 2, 1972. It should be noted that the polarity sense of the biaxial tiltmeter Y-axis was inverted with respect to mercury tube No. 2. The biaxial tiltmeter appeared to stabilize within approximately two weeks. After this time, the mean drift of the Y-axis appears to track within less than 2 μrad. The larger difference between the X-axis and mercury tube No. 1 can probably be attributed to their greater physical separation, indicating deformation in the vault floor itself. Further evidence of vault floor deformation was obtained when a borehole biaxial tiltmeter was placed in the ground near mercury tube No. 1. In this situation, the X-axis of the biaxial meter was closest to the mercury tube device and the Y-axis was now some 35 ft from the No. 2 tube. The X-axis of the biaxial tiltmeter settled down and was tracking earth tides along with the mercury tube device within a matter of a few hours. Data from this axis through the first three weeks after installation varied less than one tidal amplitude. The Y-axis in this case did not agree with the mercury tube and had a variable cyclic peak of about five tidal amplitudes every two to four days. After three weeks, the borehole tube was rotated 90° and the variable cyclic peaking immediately switched to the X-axis, whereas the Y-axis settled down immediately and tracked earth tides for the following three weeks with less than one earth tide total variation.

The source of the cyclic behavior was thus determined to be vault floor deformation but the specific forcing function was not identified.

Testing of additional Rockwell tiltmeters at the Presideo under constant temperature conditions (i.e., $<10^{-2}$C°/day) indicates instrument drift in worst case was 0.5 μrad in a two-month period.[2]

Monitoring of Seismic Events

The National Center for Earthquake Research has installed an array of Rockwell tiltmeters along a 35 km section of the San Andreas fault south of San Juan Bautista to detect regional tilt.[2] These installations were made in mid 1973. The instruments

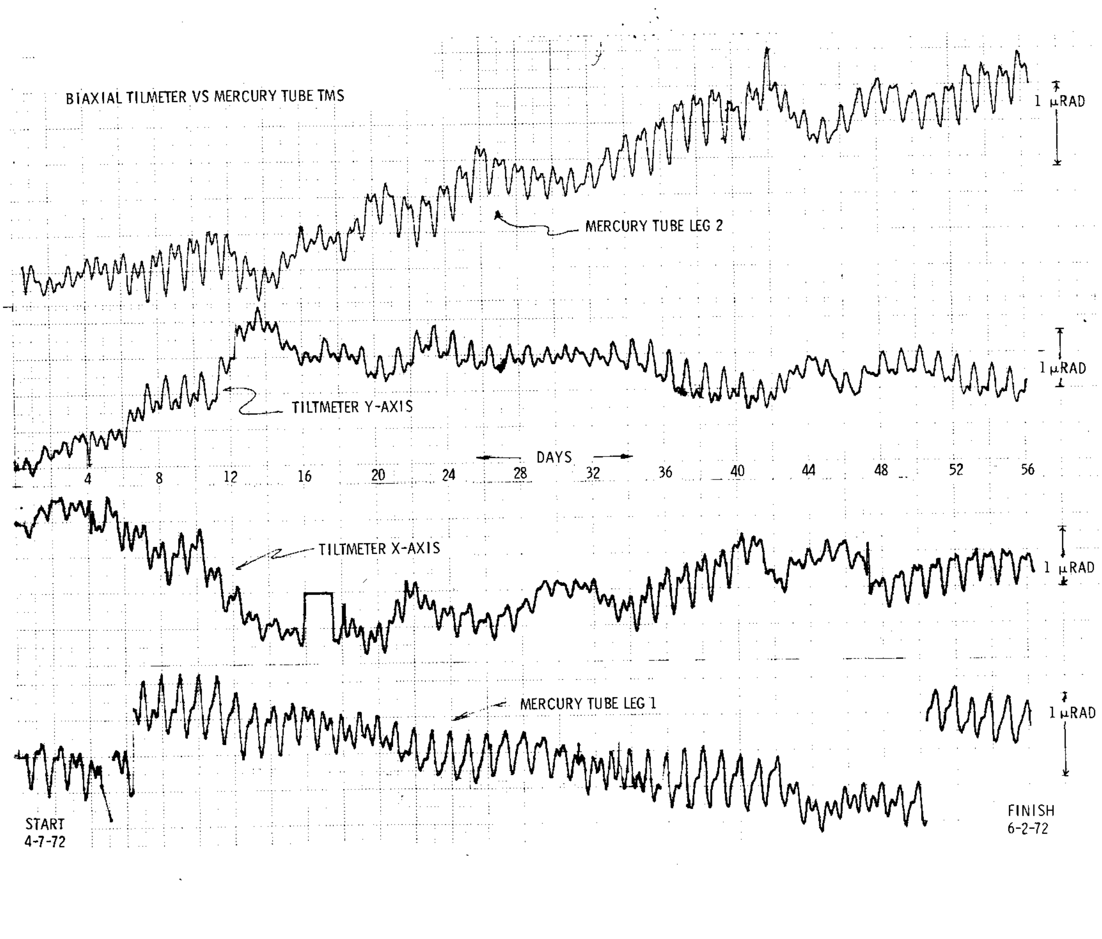

Fig. 4 Presideo vault record of biaxial tiltmeter vs mercury tube tiltmeter.

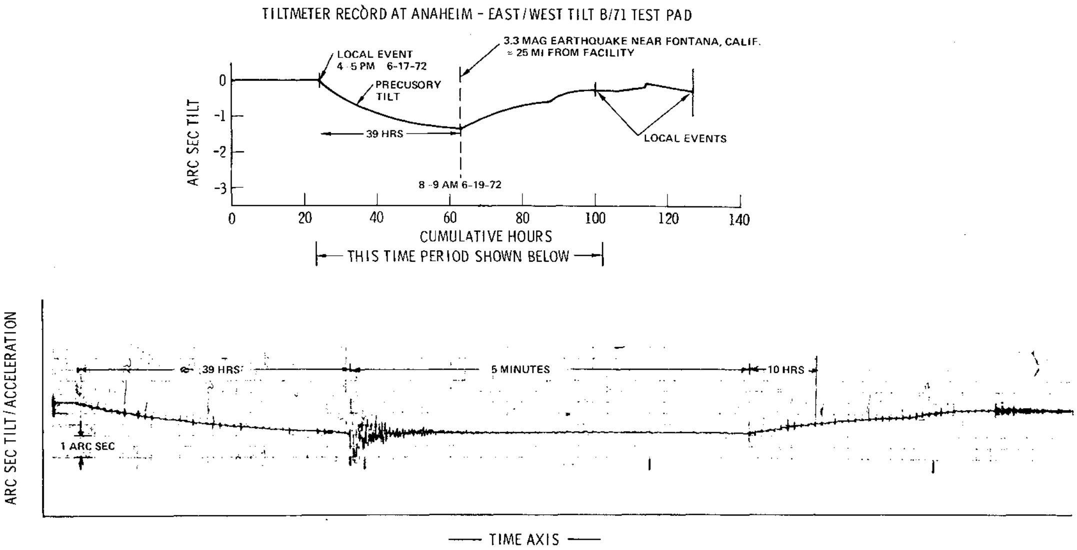

Fig. 5 Fontana, Calif. earthquake recorded at Anaheim.

are approximately 5 km apart on alternate sides of the fault and from 1–4 km from the fault. Preliminary results are consistent between the 5 southernmost instruments and show an average tilt of approximately 1.5 μrad/week generally perpendicular to the fault. The weekly tilt vector for the northernmost instrument has been systematically rotating in a clockwise direction. The U.S. Geological Survey is encouraged with the results thus far because it appears that crucial site-selection factors such as topography, meteorologic changes, and thermoelastic stresses have been kept to a minimum.

In April 1972 the California Division of Mines and Geology installed two instruments similar to those of Fig. 2 in underground silo-type constructions situated adjacent to the San Andreas fault. One instrument was in the Temblor Range one mile northeast of the fault and the other was in the Caliente Range some eight miles southwest of the fault. The vault structure consisted of two concentric culvert steel cylinders spaced 6 in. apart and buried in a 6 ft hole dug into the surrounding terrain. The floor consisted of a 4 in. layer of concrete and the opening was covered with a wooden lid over the inner cylinder and a corrugated steel lid securing the entire top of the structure. Data from the tiltmeters in these two sights were at first noisy due to large diurnal temperature effects upon the tiltmeter scale factor. Modifications to the site to reduce diurnal temperature effects consisted of a 4 in. layer of polystyrene foam sandwiched between two $\frac{3}{4}$ in. plywood subfloors located slightly above the tilt table and well below the top of the enclosure. A sealant was added around the perimeter of the top subfloor for extra insulation. Adjustments to the tilt table were made via extended PVC tubes protruding through the subfloors and capable of being manipulated to adjust the two movable tilt table legs.

The above modifications reduced the worst-case diurnal variations to 1°F. Further temperature improvement was achieved when one of the tilt table models was replaced with a borehole installation. This instrument was installed in a 4 ft deep hole at the bottom on one of the silos. The maximum diurnal temperature variation of this installation was reduced to 0.5°F. The seasonal worst-case variation noted over a period of two months was 5°F in this same downhole site. The temperature measurements were made with a thermistor installed within the borehole instrument head.

An instrument installed by the U.S. Geological Survey at the 8000 ft level in Mt. Lassen National Park has experienced output shifts from all effects of less than 0.5 μrad for a 50 day period after initial stabilization in Sept. 1972. This degree of stability would imply that adequate isolation from ambient changes can be achieved even in a worst case environment such as the top of a mountain in winter weather. The techniques used by the U.S. Geological Survey for making the borehole installations are detailed in a paper previously presented in London.[3]

The Mt. Lassen installation and three borehole instruments installed on the Kilauea Crater in Hawaii, for volcano eruption studies, have served to point out the need for EMP protection from lightning storms common in these high-altitude areas. These instruments have all received damage from lightning causing the necessity for repair and modifications. Electromagnetic shielding of the cables and resistive isolation of the input/output circuits of the electronics have proven to be adequate, as borne out by the modified instruments reinstalled in Hawaii, and still functioning in the presence of a multiplicity of lightning storms that have occurred since installation.

Data from a tiltmeter at Autonetics show several events preceded by tilts starting several hours before the actual earthquake. Clearly, the only indication that these time associated precursor tilts are geologically associated with the earthquakes that follow is their large apparent frequency of occurrence. Also of special interest is the large magnitude of some of these tilts. It can be postulated that the sandy river bed on which the Anaheim plant is located serves to amplify the seismic events through dilation of the sand or bed rock from the underground water source. Figure 5 illustrates the point being made. The predominant earthquake shown in the figure occurred six miles south of Fontana, Calif., which is within a radial distance of 25 miles from the tiltmeter installation. The apparent precursor tilt shown preceded the event by approximately 39 hr. Many of the precursors noted are not as dramatic as the changes associated with the Fig. 5 event but, nevertheless, several have been in the 1 arcsec peak range starting a day or more before the event occurs. However, it should also be noted that a few similar tilt events have occurred without the occurrence of an immediate earthquake.

Figure 6 shows another earthquake with unusual tilts preceding the event. The tilting of the earth started about 26 hr before the earthquake and had a greater tilt rate on the east–west axis. Then about 6 or 7 hr before the event, the tilt became more pronounced on the north–south axis. Note the suggested exponential tilt occurring after the event and then reversing direction about 6 hr preceding a small local event. The primary earthquake was centered near El Centro, Calif. Information obtained from the Cal Tech Seismological Lab. estimated the magnitude of the earthquake to be 4.5 on the Richter Scale.

Figure 7 shows the earthquake that occurred at Oxnard, Calif., on Feb. 21, 1973. Apparent precursor tilts on both the north–south and east–west axes starting in excess of 50 hr prior to the event are illustrated in the chart recording. The actual event triggered an electronics circuit that increased the recorder chart speed to show the high-frequency characteristics induced by the acceleration associated with the primary and secondary seismic

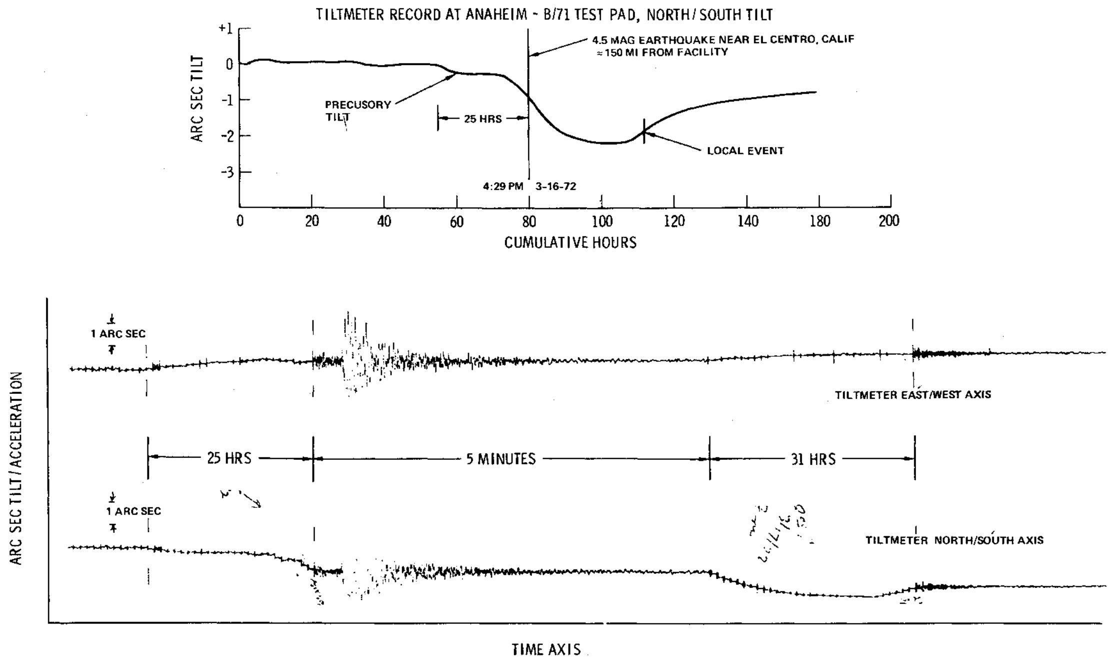

Fig. 6 El Centro, Calif. earthquake recorded at Anaheim.

waves. The difference in arrival time between the *P* and *S* waves of approximately 14 sec figured at a velocity of five miles/sec indicates that the event occurred 70 miles from the Autonetics site. This distance checks with the vector distance to the epicenter near Oxnard. It is interesting to note that the main event apparently triggered some lesser local quakes that are superimposed on the graph of the initial shockwaves. With regard to earthquake prediction, the following quote is extracted from *Science Magazine*[4]: "Combined leveling and Geodemeter surveying, accompanied by continuous monitoring of seismic activity, appears, therefore, to offer a promising basis for a long-range earthquake-warning system. It seems reasonable to hope that short-range prediction of earthquakes (on the order of hours or days) may be achieved through continuous monitoring of ground tilts, strain, seismic activity, and possibly fluctuations in the earth's magnetic field." Tiltmeter records such as depicted in Figs. 5–7 suggest that further investigation with a network for earthquake and tilt measurements be set up. Such a system is in the initial stages at the NCER at Menlo Park.

Conclusions

Data have been presented showing the sensitivity of the biaxial tiltmeter to be in the earthtide amplitude range. Furthermore, the stability of the instrument compares with that of the more cumbersome mercury tube tiltmeters. These facts suggest that the

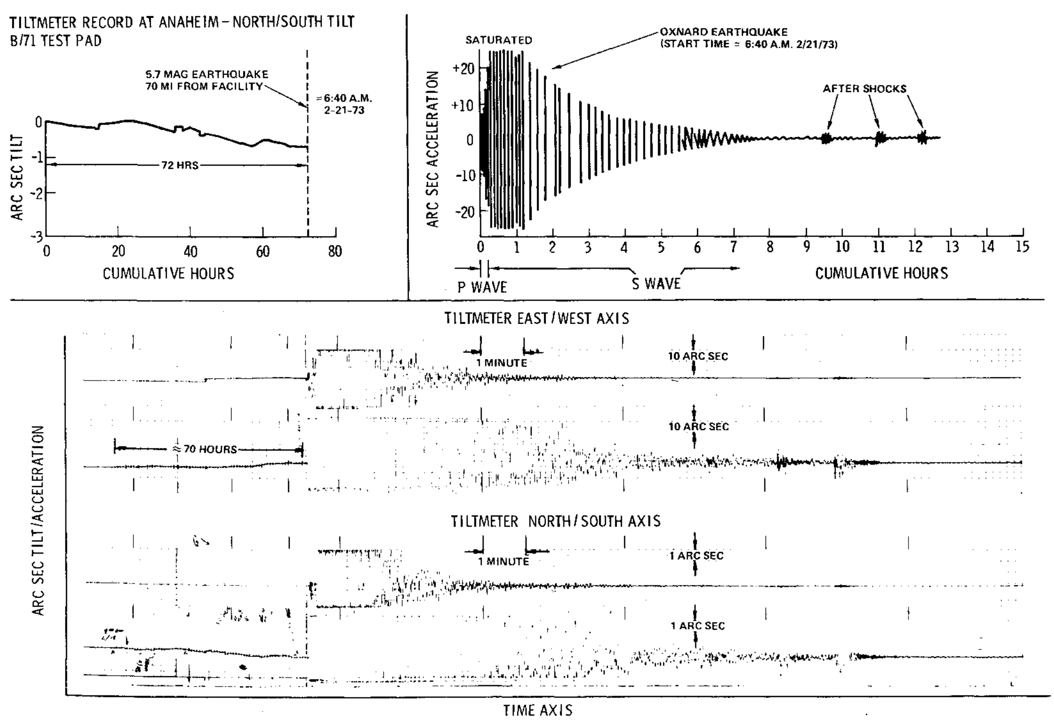

Fig. 7 Oxnard, Calif. earthquake recorded at Anaheim.

biaxial tiltmeter is useful where both accuracy and compactness is desired so that emplacement can be readily accomplished in either laboratory or remote environments. Usefulness of the instrument in the geophysical area lies in its ability to detect tilts associated with earth movement prior to earthquakes, volcano eruptions, and other seismic phenomena. In addition to tilts, the instrument's biaxial response to horizontal accelerations suggests that it may be useful as a short- and long-period seismometer.

Appendix Derivation of Random Force Function

The equation of motion of the level bubble is assumed to be

$$m(d^2/dt^2)+D(dx/dt)+Kx=F(t)$$

where

m = equivalent bubble mass, gms
D = bubble damping constant, dynes/cm/sec
K = bubble restoring constant, dynes/cm
x = deviation of bubble, cm
$F(t)$ = random force function, dynes

According to Papoulis,[5] the power spectral density of the random force function for a particle in a damping liquid is

$$S_{FF}(\omega)=2kTD$$

where

k = Boltzmann's constant, 1.38×10^{-16} ergs/°K
T = absolute temperature, °K
ω = angular frequency, rad/sec

Now the power spectra density of the deviation, x, is

$$S_{xx}(\omega)=S_{FF}(\omega)/Y^2(\omega)$$

where $Y(\omega)$ is the motional impedance function (frequency response function) or

$$S_{xx}(\omega)=\frac{S_{FF}(\omega)}{[(j\omega)^2m+j\omega D+K]^2}=\frac{2kTD}{\left[(\omega^2-\omega_0{}^2)^2+\left(\frac{D}{m}\omega\right)^2\right]m^2}$$

where

$$\omega_0=\left(\frac{K}{m}\right)^{1/2}$$

The mean square value of the position deviation is

$$\langle x^2\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{XX}(\omega)\,d\omega$$

or

$$\langle x^2\rangle=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2kTD}{\left[((\omega^2-\omega_0{}^2))^2+\frac{D}{m}\omega^2\right]m^2}d\omega$$

$$=\frac{2kTD}{\left(2\frac{D}{m}\omega_0{}^2\right)m^2}$$

$$x^2=\frac{kT}{K}$$

The restoring constant per unit distance is computed, using R as the detector radius, as

$$K=\frac{F}{x}=\frac{980m(x/R)}{x}=\frac{980m}{R}=\frac{980\times0.135}{30}=4.34\,d/\mathrm{cm}$$

The rms deviation of the bubble angle is

$$\theta_{\mathrm{rms}}=\frac{1}{R}\langle x^2\rangle^{1/2}=\frac{1}{R}\frac{kT}{K}^{1/2}=\frac{1}{30}\left(\frac{1.38\times10^{-16}\times293}{4.34}\right)^{1/2}$$

and

$$\theta_{\mathrm{rms}}=\frac{9.3\times10^{-8}}{30}\,\mathrm{cm}=\frac{9.3\times10^{-8}}{30}\,\mathrm{rad}=3.1\times10^{-9}\,\mathrm{rad}$$

Note the assumption made that the random force function operates on a solid particle. This distinction should disappear as the low-frequency end of the noise spectrum is approached.

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